TASI 2006:
Extra Dimensions
Homework 3

As discussed in lectures 3 and 4, the zero and KK modes of Z are defined by setting the Higgs vev to zero. However, due to non-zero Higgs vev, the zero and KK modes of Z mix via mass terms – kinetic terms are still diagonal. The $Z^{(0)}$-$Z^{(1)}$ (i.e., 1^{st} KK mode of Z) mass matrix is:

$$\mathcal{L}_{mass} \ni \left(Z_\mu^{(0)} Z_\mu^{(1)}\right) \begin{pmatrix} m^2 & \Delta m^2 \\ \Delta m^2 & M^2 \end{pmatrix} \begin{pmatrix} Z^{\mu\,(0)} \\ Z^{\mu\,(1)} \end{pmatrix} \tag{1}$$

where $m^2 = 1/4\ g_{Z^{(0)}}^2 v^2$, mixing term $\Delta m^2 = 1/4\ g_{Z^{(0)}} g_{5\,Z} f_1(\pi R)\, v^2$ and $M^2 = m_{KK}^2 + 1/4\ g_{5\,Z}^2 f_1^2(\pi R)\, v^2$. Here, $f_1(\pi R)$ is wavefunction of $Z^{(1)}$ evaluated at the Higgs brane ($y = \pi R$). Also, $g_{Z^{(0)}} = g_{5\,Z}/\sqrt{2\pi R + r}$ denotes coupling of $Z^{(0)}$, r is brane kinetic term at $y = 0$ and $g_{5\,Z} = \sqrt{g_{5\,2}^2 + g_5'^{\,2}}$ denotes $5D$ coupling of Z, with $g_{5\,2}$ and g_5' being the $5D$ gauge coupings of $SU(2)$ and $U(1)_Y$, respectively (assume, for simplicity, the same brane kinetic term r for all gauge fields).

Diagonalize this mass matrix, assuming $v^2/m_{KK}^2 \times$ gauge couplings $\ll 1$ where appropriate, i.e., determine

(i) the unitary transformation to go from $\left(Z^{(0)} Z^{(1)}\right)$ to physical basis and

(ii) the eigenvalues of the mass matrix.

There are 2 effects of this diagonalization.

1 Shift in Coupling of a Fermion to Z

Given couplings of a fermion to $Z^{(0)}$ and $Z^{(1)}$ (KK basis)

$$\mathcal{L}_{coupling} \ni \bar{\psi}\gamma^\mu(g, G) \begin{pmatrix} Z_\mu^{(0)} \\ Z_\mu^{(1)} \end{pmatrix} \psi \tag{2}$$

use this unitary transformation to calculate the couplings to the fermion in the physical basis, denoted by Z_{light} (which is SM Z) and Z_{heavy}.

Specifically, calculate the coupling of a fermion localized at $y = 0$ to SM Z using $g = g_{Z^{(0)}}$ and $G = g_{5\,Z} f_1(0)$ in above equation, where $f_1(0)$ is wavefunction of $Z^{(1)}$ evaluated at the fermion brane ($y = 0$).

Verify that the shift in the coupling of this fermion to Z from the zero-mode Z coupling (i.e., $g_{Z^{(0)}}$) is as shown in lecture 3: $\delta g_Z \sim g_{Z^{(0)}}^2 v^2/m_{KK}^2$, in particular, that there is *no* enhancement for large brane kinetic terms, $r/R \gg 1$.

2 Shift in Z mass

The lighter eigenvalue of mass matrix is SM Z mass. Verify that shift in Z mass from the purely zero-mode mass, i.e., $1/4 g_{Z^{(0)}}^2 v^2$, is as shown in lecture 4, in particular, that there *is* an enhancement in this shift due to $r/R \gg 1$ (when the shift is expressed in terms of $g_{Z^{(0)}}$).